Secondary Math 3H SLO Interim Review Key

Name________________________________ Period__________ Date____________

Add and subtract and simplify

1. $(6x^2 + 7x - 8) - (7x^3 + 2x^2 + 3x)$

$6x^2+7x-8-7x^3-2x^2-3x$

$\boxed{-7x^3+4x^2+4x-8}$

2. $(6x^2 + 8x - 2) + (3x^2 - 9x - 4)$

$\boxed{9x^2-x-6}$

Multiply and simplify

3. $(x - 6)(x + 2)$

$x^2+2x-6x-12$

$\boxed{x^2-4x-12}$

4. $(x^2 + x - 3)(x^2 - 3x + 5)$

$x^4-3x^3+5x^2+x^3-3x^2+5x-3x^2+9x-15$

$\boxed{x^4-2x^3-x^2+14x-15}$

Factor each expression over the complex numbers. Write your answer in factored form!

5. $x^2 - 25$

$\boxed{(x+5)(x-5)}$

6. $4x^2 - 20x + 25$

$\boxed{(2x-5)^2}$

7. $3k^2 - 24k - 60$

$3(k^2-8k-20)$

$\boxed{3(k-10)(k+2)}$

8. $6n^3 - 3n^2$

$\boxed{3n^2(2n-1)}$

9. $(5x^3 + 2x^2)(-15x - 6)$ → group

$x^2(5x+2)-3(5x+2)$

$\boxed{(x^2-3)(5x+2)}$

10. $f(x) = x^2 - 2x + 10$ use quad. form. $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$a=1, b=-2, c=10$

$x=\frac{2\pm\sqrt{(-2)^2-4(1)(10)}}{2(1)}=\frac{2\pm\sqrt{4-40}}{2}=\frac{2\pm\sqrt{-36}}{2}=\frac{2\pm 6i}{2}$

$x=1\pm 3i$

$\boxed{(x-1-3i)(x-1+3i)}$

Without graphing, determine the **number** of zeros for each of the following polynomials.

11. $f(x) = x^4 - 2x^2 - 5x + 6$

4 zeros

12. $f(x) = x^2 - 3x + 2$

2-zeros

13. $f(x) = -x^3 - x^2 - 5x - 3$

3-zeros.

Without graphing, determine the end behavior of each, then write the end behavior as a limit.

$\lim_{x\to\infty} f(x) =$ $\lim_{x\to-\infty} f(x) =$

14. $f(x) = 2x^2 - 8x + 6$

$\lim_{x\to\infty} f(x)=\infty$, $\lim_{x\to-\infty} f(x)=\infty$

15. $f(x) = -x^2 + 9x$

$\lim_{x\to\infty} f(x)=-\infty$ $\lim_{x\to-\infty} f(x)=-\infty$

16. $f(x) = -x^3 - x^2 - 3$

$\lim_{x\to\infty} f(x)=-\infty$

$\lim_{x\to-\infty} f(x)=\infty$

17. $f(x) = x^3 - 3x + 2$

$\lim_{x\to\infty} f(x)=\infty$, $\lim_{x\to-\infty} f(x)=-\infty$

18. $f(x) = x^5 - 3x$

$\lim_{x\to\infty} f(x)=\infty$, $\lim_{x\to-\infty} f(x)=-\infty$

For the given polynomials determine which of the binomials listed are factors.

19. $f(x) = x^3 - x^2 - 5x - 3$
a. x+1 $f(-1)=0$ yes
b. x-1 $f(1)=-8$ no
c. x -3 $f(3)=0$ yes

20. $f(x) = x^3 - 3x + 2$
a. x-1 $f(1)=0$ yes
b. x+2 $f(-2)=0$ yes
c. x+1 $f(-1)=4$ no

21. $f(x) = x^3 - 3x$
a. x+0 $f(0)=0$ yes
b. x-2 $f(2)=2$ no
c. x-1 $f(1)=-2$ no

Identify the zeros, their multiplicity, determine whether they touch or cross the x-axis at each zero → $(x+3)(x-3)$

22. $f(x) = (x + 1)(x - 1)(x - 3)$

Zeros	mult.	touch/cross
(-1,0)	1	cross
(1,0)	1	cross
(3,0)	1	cross

23. $f(x) = 6x^3(x^2 - 9)(x + 2)$

Zeros	mult.	touch/cross
(0,0)	3	cross
(3,0)	1	cross
(-3,0)	1	cross
(-2,0)	1	cross

The sequence is geometric. Find a) the common ratio, b) the eighth term, c) a recursive rule for the *n*th term, and d) an explicit rule for the *n*th term. Show work!

40. 2, 6, 18, 54

a) $r=3$

b) $a_8 = 2\cdot 3^{8-1} = 2\cdot 3^7 = 4374$

c) $a_1 = 2,\ a_n = 3\cdot a_{n-1}$

d) $a_n = 2\cdot 3^{n-1}$

41. Find an algebraic expression for h(x) using the given functions. Simplify if possible. Determine the domain, in interval notation, for each *a, b, c, and d.*

$f(x) = x^2 + 3x - 4$ and $g(x) = 2x + 1$

a. $h(x) = (f+g)(x)$

$(x^2+3x-4)+(2x+1) = \boxed{x^2+5x-3 \quad D:(-\infty,\infty)}$

b. $h(x) = (f-g)(x)$ $(x^2+3x-4)-(2x+1)$

$x^2+3x-4-2x-1 = \boxed{x^2+x-5 \quad D:(-\infty,\infty)}$

c. $h(x) = (fg)(x)$

$(x^2+3x-4)(2x+1)$

$2x^3+x^2+6x^2+3x-8x-4 = \boxed{2x^3+7x^2-5x-4 \quad D:(-\infty,\infty)}$

d. $h(x) = \left(\frac{f}{g}\right)(x)$

$\boxed{\dfrac{x^2+3x-4}{2x+1} \quad D:(-\infty,-\tfrac{1}{2})\cup(-\tfrac{1}{2},\infty)}$

42. Evaluate each of the following given $f(x) = x^2 + 3x - 4$ and $g(x) = 2x + 1$

a. $f(-3)\cdot g(2)$

$f(-3) = (-3)^2+3(-3)-4$
$= 9-9-4 = -4$
$g(2) = 2(2)+1 = 5$
$-4\cdot 5 = \boxed{-20}$

b. $f(2) + g(1)$

$f(2) = (2)^2+3(2)-4$
$= 4+6-4 = 6$
$g(1) = 2(1)+1 = 3$
$6+3 = \boxed{9}$

c. $f(0) - g(-2)$

$f(0) = (0)^2+3(0)-4 = -4$
$g(-2) = 2(-2)+1 = -3$
$-4-(-3) = -4+3 = \boxed{-1}$

43. Determine if the functions are even, odd or neither.

a. $f(x) = -3x^4$

$f(-x) = -3(-x)^4$
$= -3x^4$ same
even

b. $f(x) = 2x^3 + 5x$

$f(-x) = 2(-x)^3+5(-x)$
$= -2x^3-5x$
$-f(x) = -2x^3-5x$ same
odd

c. $f(x) = 6x^3 - x^2$

$f(-x) = 6(-x)^3-(-x)^2$
$= -6x^3-x^2$
$-f(x) = -6x^3+x^2$
neither.

44. Find the average rate of change for each function on the specified interval.

$f(x) = 4x^2 + 12x + 9$ *on* $[-3,0]$

$f(-3) = 4(-3)^2+12(-3)+9$
$= 36-36+9 = 9$

$f(0) = 4(0)^2+12(0)+9 = 9$

$(-3, 9)$
$(0, 9)$

$\dfrac{9-9}{0-(-3)} = \dfrac{0}{3} = \boxed{0}$

45. Find the domain, the vertical asymptote(s) and the horizontal asymptote of $\frac{x-5}{(x-9)(x+6)}$.

$\frac{x}{x^2} \to 0$

$x \neq 9 \quad x \neq -6$

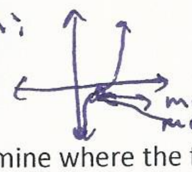
Domain: $(-\infty,-6)\cup(-6,9)\cup(9,\infty)$

VA: $x=-6,\ x=9$

HA: $y=0$

46. Determine the local maximums and local minimums of $f(x) = (x-2)^3 - 1$

Graph:



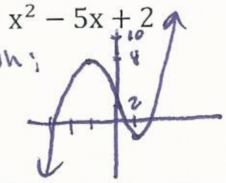
min at (1.91, −1.00)
max at (2.13, −.998)

(small dip in graph.)

47. Determine where the following function is increasing, decreasing and constant.

$f(x) = x^3 + x^2 - 5x + 2$

graph:



Increasing $(-\infty, -1.6\overline{6})$ $(1,\infty)$

Decreasing $(-1.6\overline{6}, 1)$

48. Solve the following functions for the specified variable.

a. $\frac{1}{c} - \frac{c}{a^2-b^2} = 0$ *for c*

$\frac{1}{c} = \frac{c}{a^2-b^2}$

$\boxed{c^2 = \sqrt{a^2-b^2}}$

$\boxed{c = \pm\sqrt{a^2-b^2}}$

b. $\frac{r_1}{r_2} = \sqrt{\frac{M_2}{M_1}}$ *solve for* M_1

$\left(\frac{r_1}{r_2}\right)^2 = \left(\sqrt{\frac{m_2}{m_1}}\right)^2$

$\frac{r_1^2}{r_2^2} = \frac{m_2}{m_1}$

$\frac{r_1^2 m_1}{r_1^2} = \frac{r_2^2 m_2}{r_1^2}$

$\boxed{M_1 = \frac{r_2^2 M_2}{r_1^2}}$

Use long division to rewrite the expression.

24. $\frac{2x^4-x^3-4x-2}{x^2+x+1}$

$$\begin{array}{r} 2x^2-3x+1 \\ x^2+x+1 \overline{) 2x^4-x^3+0x^2-4x-2} \\ -2x^4 \mp 2x^3 \mp 2x^2 \\ -3x^3-2x^2-4x \\ \pm 3x^3 \pm 3x^2 \pm 3x \\ x^2-x-2 \\ -x^2 \mp x \mp 1 \\ -2x-3 \end{array}$$

$\boxed{2x^2-3x+1+\frac{-2x-3}{x^2+x+1}}$

25. $\frac{x^3+2x^2-3x+4}{x+1}$

$$\begin{array}{r} x^2+x-4 \\ x+1 \overline{) x^3+2x^2-3x+4} \\ -x^3 - x^2 \\ x^2-3x \\ -x^2 - x \\ -4x+4 \\ \pm 4x \pm 4 \\ 8 \end{array}$$

$\boxed{x^2+x-4+\frac{8}{x+1}}$

Perform the indicated operation. Simplify if possible.

26. $\frac{6y^2}{5x^2} \div \frac{3y^2}{4x^6}$

$\frac{{}^{2}\cancel{6y^2}}{5\cancel{x^2}} \cdot \frac{4x^{\cancel{6}\,4}}{{}^{1}\cancel{3y^2}} = \boxed{\frac{8x^4}{5}}$

27. $\frac{x+5}{x-6} \cdot \frac{2x-12}{x^2-25}$

$\frac{\cancel{x+5}}{\cancel{x-6}} \cdot \frac{2\cancel{(x-6)}}{\cancel{(x+5)}(x-5)}$ $\boxed{\frac{2}{x-5}}$

28. $\frac{x^2+5x-14}{3x^3-6x^2} \cdot \frac{2x^2+6x}{x^2+10x+21}$

$\frac{\cancel{(x+7)}\cancel{(x-2)}}{3x^2\cancel{(x-2)}} \cdot \frac{2x\cancel{(x+3)}}{\cancel{(x+7)}\cancel{(x+3)}}$ $\boxed{\frac{2}{3x}}$

29. $\frac{2x}{x+5}\frac{(x+8)}{(x+8)} - \frac{x}{x+8}\frac{(x+5)}{(x+5)}$

$\frac{2x(x+8)-x(x+5)}{(x+5)(x+8)} = \frac{2x^2+16x-x^2-5x}{(x+5)(x+8)} = \boxed{\frac{x^2+11x}{(x+5)(x+8)}}$

30. $\frac{-6}{x-3}\frac{(x-2)}{(x-2)} + \frac{5}{x-2}\frac{(x-3)}{(x-3)}$

$\frac{-6(x-2)+5(x-3)}{(x-3)(x-2)}$

$\frac{-6x+12+5x-15}{(x-3)(x-2)} = \boxed{\frac{-x-3}{(x-3)(x-2)}}$

31. $\frac{3}{x^2-3x+2} - \frac{3}{x-2}\frac{(x-1)}{(x-1)}$

$(x-2)(x-1)$

$= \frac{3-3(x-1)}{(x-2)(x-1)}$

$= \frac{3-3x+3}{(x-2)(x-1)} = \frac{-3x+6}{(x-2)(x-1)} = \frac{-3\cancel{(x-2)}}{\cancel{(x-2)}(x-1)}$

$= \boxed{\frac{-3}{x-1}}$

32. $\frac{x+2}{x^2+9x+20} + \frac{x-1}{x^2+x-12}$

$(x-3)(x+4)(x+5)$ $\quad (x+4)(x-3)(x+5)$

$\frac{(x-3)(x+2)+(x-1)(x+5)}{(x-3)(x+4)(x+5)} = \frac{x^2-x-6+x^2+4x-5}{(x-3)(x+4)(x+5)} = \boxed{\frac{2x^2+3x-11}{(x-3)(x+4)(x+5)}}$

Solve the equation.

33. $\left[\frac{3}{x+2} + \frac{6}{x^2+2x} = \frac{3-x}{x}\right]\frac{x(x+2)}{1}$

$x^2+2x = x(x+2)$

$x \neq 0$, $x \neq -2$

$3x+6 = (3-x)(x+2)$

$3x+6 = -x^2+x+6 \rightarrow x^2+2x=0$

$x(x+2)=0$

$x=0$ $x=-2$

no solution

34. $\left[\frac{4x}{x+4} + \frac{5}{x-1} = \frac{15}{x^2+3x-4}\right]\frac{(x+4)(x-1)}{1}$

$x^2+3x-4 = (x+4)(x-1)$

$4x(x-1)+5(x+4)=15$

$4x^2-4x+5x+20=15 \rightarrow 4x^2+x+5=0$, use quad. form

$x = \frac{-1\pm\sqrt{(1)^2-4(4)(5)}}{2(4)} = \boxed{\frac{-1\pm i\sqrt{79}}{8}}$

35. $\sqrt{4x-23} - 3 = 2$ $+3$ $+3$

$(\sqrt{4x-23})^2 = (5)^2$

$4x-23=25$

$4x=48$

$\boxed{x=12}$

36. $\sqrt{2x+3} - 7 = 0$ $+7$ $+7$

$(\sqrt{2x+3})^2 = (7)^2$

$2x+3=49$

$2x=46$

$\boxed{x=23}$

Find the coefficient of the given term in the binomial expansion.

37. $x^{11}y^3$ term, $(x+y)^{14}$

$\binom{14}{11} = \boxed{364}$

The sequence is arithmetic. Find a) the common difference, b) the tenth term, c) a recursive rule for the *n*th term, and d) an explicit rule for the *n*th term. Show work!

38. 6,10, 14, 18, …

a) $d=4$

b) $a_{10} = 6+(10-1)(4) = 42$

c) $a_1=6$, $a_n = a_{n-1}+4$

d) $a_n = 6+(n-1)4 = 6+4n-4 = 4n+2$

$\boxed{a_n = 4n+2}$

39. An auditorium has 30 rows with 10 seats in the first row, 12 seats in the second row, 14 seats in the third row, and so forth. How many seats are in the auditorium?

$n=30$ $\quad a_1=10$ $\quad a_2=12$, $d=2$

$S = \frac{n}{2}(2a_1+(n-1)d) = \frac{30}{2}(2(10)+(30-1)(2))$

$= 15(20+58) = 15(78) = \boxed{1170 \text{ seats}}$

49. Describe the sequence of transformations that produce the graph of the given function.
$f(x) = x^2$ and $g(x) = (x+4)^2 + 5$

Horizontal shift left 4, vertical shift up 5.

50. Sam receives $40,000 the first year of his new job. He is guaranteed a 1.5% increase per year for each year after that. What is Sam's total salary if he works at this job for 5 years?

$S \rightarrow$ total salary

$t \rightarrow$ time - in years worked

Geometric sequence.

$a_1 = 40{,}000$

$r = 1.5\% = .015 + 1 = 1.015$

$a_5 = 40{,}000 \cdot (1.015)^{5-1}$

$= \$42{,}454.54$